Color Image Segmentation Using Relation of Equivalence

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Abstract. An algorithm for color image segmentation is defined and implemented. It uses a binary relation of equivalence that is established with respect to different metrics defined in topological spaces. The designed algorithm works with RGB represented color images in BMP format. The algorithm is based on relating two points $\vec{x}$, $\vec{y}$ in a space or RGB image if and only if it is fulfilled that a metric d($\vec{x}, \vec{y}$) is less or equal to a threshold $r>0$. Previously to the application of the relation of equivalence one fixed reference point is calculated on the expectation of a selected set of points. The posterior realized operations are dealing with different types of metrics in which different interesting results are observed comparing to other segmentation algorithms for this kind of images. The presented algorithm may be used to process satellite multichannel image data.

Key words: image segmentation, metric.

1 Introduction

The necessity to measure is evident in the majority of the technical and scientific activities. However, it is not interesting only to count with measurements, but also to know if these measurements are valid. To this end, one must remember the definition of measurement as the "process for which they are assigned numbers or symbols for attributes of entities from real world in such a manner that describes them according to the clearly defined rules" [1]. An assignment established between the real world and measurement values is usually denominated measuring scale.

The abstract concept of a metric space was introduced in 1906 by French mathematic M. Fréchet, and then was developed by German mathematic Felix Hausdorff. The Soviet mathematic J. V. Smirnov found the conditions necessary and suffiecient for the metricsability of a topologic space. Actually, the metric spaces are a particular case of the topological spaces.

In our application the segmentación is performed using the vectors of RGB colors on the base of a Cartesian coordinate system where each of its primary spectral components are red, green and blue. Hence, the images of RGB model consist of three planes of independent images, one for each primary color. Thus, the usage of RGB model for image processing has a sense when the proper images are expressed in a

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Industrial Informatics
Research in Computing Science 31, 2007, pp. 183-192

natural form in terms of three color planes. This condition permits that RGB images are employed in applications of multichannel data processing from airborne or spaceborne sensors.

The presented algorithm is defined on the pixels of a RGB image, which for entrance is considered as a metric space, and a formal tool named relation of equivalence that induce a partition on the metric. To develop the algorithm, in continuation important concepts y notations are briefly considered.

2 Theoretical aspects

Let us define a binary relation on a set $X=\{\vec{x} / \vec{x}$ is a point in RGB space$\}$ as following:

$$\vec{x} R \vec{y} \leftrightarrow d(\vec{x},\vec{y}) \leq r, r>0 \tag{1}$$

Clearly, R is reflexive, symmetric and transitive on metric space (X, d); to prove these properties one can use the properties that by definition a metric must fulfill [2]:

$$\begin{aligned} &\text{a) } d(\vec{x},\vec{y}) \geq 0;\ d(\vec{x},\vec{y})=0 \leftrightarrow \vec{x}=\vec{y} \\ &\text{b) } d(\vec{x},\vec{y}) = d(\vec{y},\vec{x}) \\ &\text{c) } d(\vec{x},\vec{y}) \leq d(\vec{x},\vec{z}) + d(\vec{z},\vec{y}) \end{aligned} \tag{2}$$

Proof. Let $\vec{x} \in X$; by item a) from 2) one have that $d(\vec{x},\vec{x})=0 \leq r \rightarrow \vec{x} R \vec{x}$ $\forall \vec{x} \in X$, say R is reflexive. Let $\vec{x} R \vec{y} \rightarrow d(\vec{x},\vec{y}) \leq r$ and by item b) from 2) one have that $d(\vec{x},\vec{y}) = d(\vec{y},\vec{x})$. Hence, $d(\vec{y},\vec{x}) \leq r \rightarrow \vec{y} R \vec{x}$, say R is symmetric. Let $\vec{x} R \vec{z}$ y $\vec{z} R \vec{y} \rightarrow d(\vec{x},\vec{z}) \leq r_1$ and $d(\vec{z},\vec{y}) \leq r_2$, and by item c) from 2) one have that $d(\vec{x},\vec{y}) \leq d(\vec{x},\vec{z}) + d(\vec{z},\vec{y}) \leq r_1 + r_2 = r \rightarrow \vec{x} R \vec{z}$, say R is transitive. Thus, R is a binary relation of equivalence.

It is legal to measure in different manners with metrics both from the practical and mathematical points of view, because it corresponds to the general notion of distance. As it was before mentioned, different metrics [3] were employed in the algorithm, such as:

- City-block metric:

$$d_1(\vec{x},\vec{y}) = |x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3| \tag{3}$$

- Euclidean metric:

$$d_2(\vec{x},\vec{y}) = [(x_1-y_1)^2 + (x_2-y_2)^2 + (x_3-y_3)^2]^{1/2} \tag{4}$$

- Chessboard metric:

$$d_\infty(\vec{x},\vec{y}) = \max\{|x_1-y_1|, |x_2-y_2|, |x_3-y_3|\} \tag{5}$$

A relation exists between these metrics in the following manner:

$$d_\infty(\vec{x},\vec{y}) \le d_2(\vec{x},\vec{y}) \le d_1(\vec{x},\vec{y}) \qquad (6)$$

The above described let us state that it is independent what metric is used in the application. However, in practice it is not true.

After the anterior definitions, we can start properly to design the topologic properties on the space (X, d). For each $\vec{x}_0$ that belongs to X we define:

$$\text{Open sphere: } B(\vec{x}_0, r) = \{\vec{x} / d(\vec{x}_0, \vec{x}) < r\} \qquad (7)$$

$$\text{Closed sphere: } \overline{B}(\vec{x}_0, r) = \{\vec{x} / d(\vec{x}_0, \vec{x}) \le r\} \qquad (8)$$

where d is any of defined before metrics.

In Fig. 1 shows graphically the relation between the unitary spheres, say r=1, for the different metrics d_1, d_2 y d_∞.

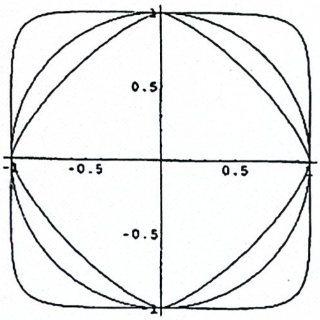

Fig.1 Relation between unitary spheres

Also, the first thing that one have to determine to carry out the application is calculate the point $\vec{x}_0$. It can be obtained calculating the expectation [5]:

$$\langle \vec{x}_0 \rangle = \sum_{i=0}^{N} \frac{x_i}{N} \qquad (9)$$

3 Proposed segmentation algorithm

To process RGB images each point x that belong to X stays in a system of three-dimensional Cartesian coordinates and is denoted as:

$$\vec{x} = (\text{Red, Green, Blue}) = (x_R, x_G, x_B)$$

The segmentation algorithm of an RGB image [4], consists in dividing it according to a partition that induces the binary relation of equivalence R defined by (1). Each partition is constitutes by a region of common features that depend on the selection or subset of a set Y, y on fundamental problem that consists in finding the optimal threshold value r. Consider the original image in Fig. 2:

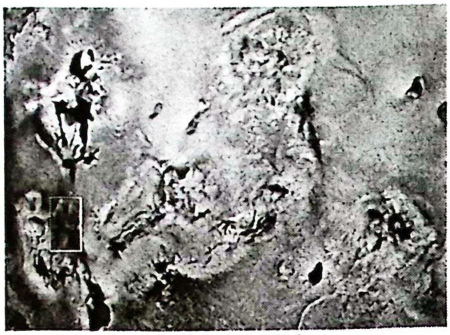

Fig. 2 Original RGB image

Using equation (9) one can obtain the expectation for each of its primary components x_R, x_G y x_B of a subset Y from X and determine $\vec{x}_0$. In Fig. 2 the considered subset Y is marked as a white rectangle. As it was mentioned above, the RGB images consist of three planes of independent images, one for each primary color [2] then the expectations are calculated for each of them:

$$\langle x_0 \rangle = \sum_{i=0}^{N} \frac{x_{R_i}}{N} \tag{10}$$

$$\langle y_0 \rangle = \sum_{i=0}^{N} \frac{y_{G_i}}{N} \tag{11}$$

$$\langle z_0 \rangle = \sum_{i=0}^{N} \frac{z_{B_i}}{N} \tag{12}$$

Once the expectations are calculated for each components of $\vec{x}_0$, i.e. (10), (11) and (12), we have that

$$\vec{x}_0 = (x_0, y_0, z_0)$$

Considering the image in Fig. 2 and applying the definition (8) together with the use of the metric $d_2(\vec{x}, \vec{y})$ with respect to the point $\vec{x}_0$ and calculating with respect to the selected subset Y, we have that one of the partitions or class $\overline{B}_2(\vec{x}_0, r) = \{\vec{x} / d_2(\vec{x}_0, \vec{x}) \leq r\}$ of the image in Fig. 2 for r=44 is:

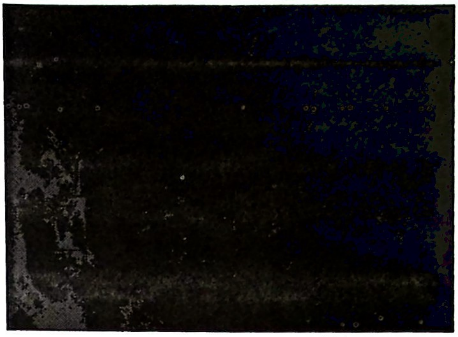

Fig. 3 Image of the class or sphere $\overline{B}_2(\vec{x}_0, \mathrm{r})$

Let us now apply in the same manner the same process with the same threshold r=44 to different images shown in Fig. 4, Fig. 5 but changing the metric.

Fig. 4 Original RGB image

Now consider the image in Fig. 4. Applying the definition (8), together with the use of the metric $d_\infty(\vec{x}, \vec{y})$ with respect to the point $\vec{x}_0$ calculated with respect to the selected subset Y, we have that one of the partitions or class $\overline{B}_\infty(\vec{x}_0, \mathrm{r}) = \{\vec{x} / d_\infty(\vec{x}_0, \vec{x}) \leq \mathrm{r}\}$ from the image in Fig. 2 for r=44 is:

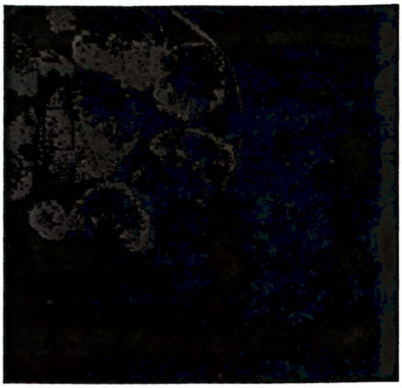

Fig. 5 Image of the class or sphere $\overline{B}_{\infty}(\vec{x}_0, r)$.

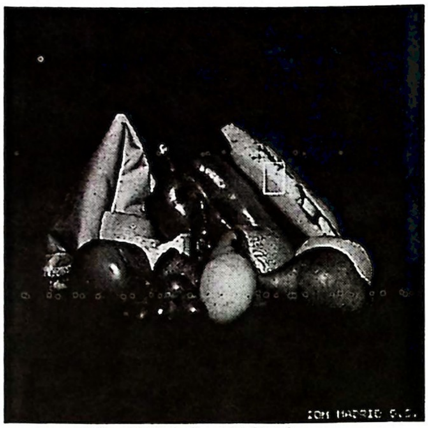

Fig. 6 Original RGB image

Finally, consider the image in Fig. 6. Applying the definition (8) together with the use of the metric $d_1(\vec{x}, \vec{y})$ with respect to the point $\vec{x}_0$ calculated with respect to the selected subset Y, we have that one of the partitions or class $\overline{B}_1(\vec{x}_0, r) = \{\vec{x} \,/\, d_1(\vec{x}_0, \vec{x}) \leq r\}$ of the image in Fig. 2 for r=44 is:

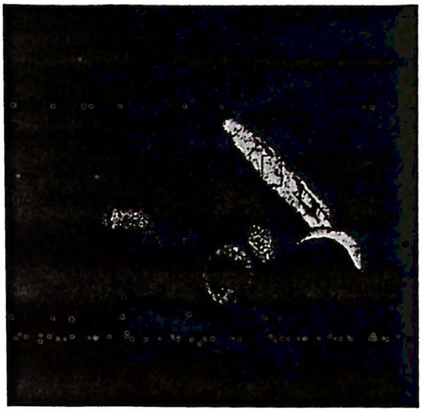

Fig. 7 Image of the class or sphere $\overline{B}_1(\vec{x}_0, r)$.

Comparing the images in Fig. 3, Fig. 5 and Fig. 7 it is clearly observed that the image in Fg.5 is defined better. In this case, the metric $d_\infty(\vec{x}, \vec{y})$ was used. Then it follows Fig. 3 with the metric $d_2(\vec{x}, \vec{y})$ and, finally, Fig.7 where $d_1(\vec{x}, \vec{y})$ was used.

4 Implementation

The presented algorithm was implemented in C++ and was applied to the common test images. Image analysis consists of image segmentation. The segmentation subdivides an image in its parts that it constitute of and objects that it conforms. Such a subdivision is performed until a level predetermined by the practical problem is reached, i.e., the segmentation must stop when the objects of interest for the particular application have been separated [9]. Steps to follow for image segmentation with the method of the binary relation of equivalence on X [10] defined en terms of a metric are:

- Select the image to analyze.
- Get a subset Y of points $\vec{x}$ of X (white rectangle that appear at the image).
- Determine the point $\vec{x}_0$ from the subset Y, and using the relations for the expectation (10), (11) and (12), determine $\vec{x}_0$.
- Select one of the metrics defined by (3) or (4) or (5).
- Define the threshold r.
- Apply the definition of the closed sphere (8), where the selected metric is compared wit respect to the threshold r having $\vec{x}_0$ as a fixed point.

5 Experimental Results

The described in the previous section algorithm were tested on a set of 512x512 images shown in Fig. 2, Fig. 4, Fig.6. In this paper we present the experimental results of the proposed algorithm for case when it was applied to each image changing the metric with a fixed threshold r=44. The results are observed in the following Table where as a measure of segmentation a function of normalized mean defined $\varphi : \mathrm{R}^3 \rightarrow \mathrm{R}$ is taken:

$$\varphi(\overrightarrow{x_0}) = \sum_{i=0}^{M} \frac{\rho(\overrightarrow{\mathrm{x_i}})}{M} \tag{13}$$

where M=(512)x(512) and $\rho : \mathrm{B}(\overrightarrow{\mathrm{x_0}}, \mathrm{r}) \rightarrow N$, defined as

$$\rho(\overrightarrow{\mathrm{x_i}}) = \begin{cases} \rho(\overrightarrow{\mathrm{x_{i-1}}}) + 1, \text{ if } & \overrightarrow{\mathrm{x_i}} \in \mathrm{B}(\overrightarrow{\mathrm{x_0}}, \mathrm{r}) \\ 0, \text{ otherwize} \end{cases}; \quad i=1,2,\ldots,M \tag{14}$$

and $\rho(\overrightarrow{\mathrm{x_0}}) = 0$.

Table. Normalized mean function.

$\varphi(\overrightarrow{x_0})$	Images		
	Fig. 2	Fig. 4	Fig.6
Unitary metric: $\mathrm{d_1}(\vec{x}, \vec{y})$	0.013385	0.104860	0.023594
Euclidian metric: $\mathrm{d_2}(\vec{x}, \vec{y})$	0.008249	0.068844	0.015865
Cubic metric: $\mathrm{D_\infty}(\vec{x}, \vec{y})$	0.019259	0.098743	0.019767

6 Conclusions

Utilizing the proposed algorithm to realize the analysis of RGB images, employing a binary relation of equivalence defined in terms of some metric, one have to solve the problem or limit of the dependence of two parameters. One of these parameters have to be found averaging ($\vec{x}_0$) and another one, (r), in a random manner. Next, in practice the type of the used metric influences as well, until the objects are separated or until the problem of recognition is solved in some form. The latter is very subjective because depends on the observer of the segmented object. However, in the various presented examples the way to determine the parameter $\vec{x}_0$ was found. It was found that the threshold value r=44 results in the desired segmentation for our particular application of recognition of forms and image analysis. Observing the Table, one can conclude that increasing the value of $\varphi(\overrightarrow{x_0})$, the segmentation of the analyzed image become to be better. Such a behavior depends only on the employed metric, because the threshold value was fixed for all metrics.

Acknowledgements

This work was supported by Instituto Politécnico Nacional as a part of the research project SIP#20071380.

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